Microeconomics II

2nd lecture

(Online version)

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Pure strategy Nash equilibrium

Definition 7.7: Pure Strategy Nash Equilibrium

Given a strategic form game $G = (S_i, u_i)_{i=1}^N$, the joint strategy $\hat{s} \in S$ is a *pure strategy Nash equilibrium* of G if for each player i,

$$u_i(\hat{s}) \geq u_i(s_i, \hat{s}_{-i})$$

for all $s_i \in S_i$.

- The dominant strategy equilibrium is a pure strategy Nash equilibrium

	L	R
U	3, 0	0, −4
D	2, 4	−1, 8

Mixed strategies

- Given a strategic form game $G = (S_i, u_i)_{i=1}^N$, *a mixed strategy*, m_i for player i is a probability distribution over S_i. That is, $m_i: S_i \rightarrow [0,1]$ assigns to each s_i the probability $m_i(s_i)$ that s_i is played.

$$M_i = \{m_i: S_i \rightarrow [0,1] \mid \textstyle\sum_{s_i \in S_i} m_i(s_i) = 1\}$$

is the set of mixed strategies for player i.

Nash equilibrium

- Let $M = \times_{i=1}^N M_i$ be the set of joint mixed strategies. For a joint mixed strategies $m_i \in M_i$, $u_i(m)$ for each player *i* is

$$u_i(m) = \sum_{s \in S} m_1(s_1) \cdots m_N(s_N) u_i(s).$$

- **Definition:** A joint strategy $\widehat{m} \in M$ is a Nash equilibrium of G if for each player i,

$$u_i(\widehat{m}) \geq u_i(m_i, \widehat{m}_{-i})$$

for all $m_i \in M_i$.

  - Note that a pure strategy Nash equilibrium is a Nash equilibrium such that each $\hat{s}_i$ is played with probability 1

Multiple Nash Equilibria: "Battle of Sexes"

- Both desperately wanted to be together in an event, but
  - Ryu preferred Richard Strauss to Dalai Lama
  - His wife preferred Dala Lama to Richard Strauss

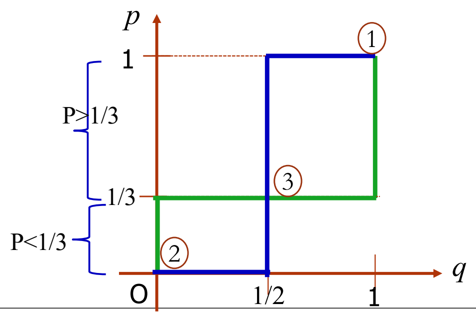
		His wife	
		Opera (*Der Rosenkavalier*)	Dalai Lama's Public Lecture
Ryu	Opera (*Der Rosenkavalier*)	2, 1	0, 0
	Dalai Lama's Public Lecture	0, 0	1, 2

And there's a mixed strategy equilibrium!

How to find mixed strategy Nash equilibrium

- Let p and $1\text{-}p$ be probabilities that the wife goes to Opera and Dalai Lama's lecture, respectively.
- Also let q and $1\text{-}q$ be probabilities that Ryu goes to Opera and Dalai Lama's lecture, respectively.
- **<u>When Ryu goes to Opera</u>**, his expected payoff is: $2\times p + 0\times(1-p) = 2p$.
- **<u>When Ryu goes to the lecture</u>**, his expected payoff is: $0\times p + 1\times(1-p) = 1-p$.
  - If $2p > 1\text{-}p$ (which is equivalent to $p>1/3$), then Ryu should go to Opera. That is $q=1$ (recall q is the probability he goes to Opera).
  - Otherwise but except $p=1/3$, he should go to the lecture. That is $q = 0$.
  - When $p=1/3$, he has no preference of the choice.
- In the same way, his wife's choice is also changeable depending on q ⪋ ½.

	His wife	
Ryu	Opera (*Der Rosenkavalier*)	Dalai Lama's Public Lecture
Opera (*Der Rosenkavalier*)	2, 1	0, 0
Dalai Lama's Public Lecture	0, 0	1, 2



Properties of Nash Equilibrium

- As seen in the previous example, Nash equilibrium has the following properties:

1. The equilibrium is not always Pareto/social optimal;
   - The dominant strategy equilibrium is Nash equilibrium.
2. Once players reach the equilibrium, they have no incentive to deviate from the equilibrium action;
3. Pure strategy Nash equilibrium does not necessarily exist.
   - We extend it into *the mixed strategy Nash equilibrium* to guarantee its existence.
4. The Nash equilibrium may be multiple.
   - Then, the equilibrium does not provide the winning strategy.

Simplified Nash Equilibrium tests

- The following statements are equivalent:

1. $\widehat{m} \in M$ is a Nash equilibrium;
2. For every player i, $u_i(\widehat{m}) = u_i(s_i, \widehat{m}_{-i})$ for every $s_i \in S_i$ given positive weight by $\widehat{m_i}$, and $u_i(\widehat{m}) \geq u_i(s_i, \widehat{m}_{-i})$ for every $s_i \in S_i$ given zero weight by $\widehat{m_i}$;
3. For every player i, $u_i(\widehat{m}) \geq u_i(s_i, \widehat{m}_{-i})$ for every $s_i \in S_i$.