Transcription: 2nd class: Introduction

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I believe you have already read the reading assignment. You may feel it is quite abstract. So, I explain these notions by using examples.

Please see page 2, the bottom of the first page of the handout. Here, I copy the definition of Nash equilibrium in the textbook. The point of this definition is, under the equilibrium, no player has incentive to deviate the equilibrium action because the deviation does not yield higher payoff.

To find pure strategy Nash equilibrium, we need to find the action which yields the higher payoff in considering others' actions. Consider others' action, each of the players needs to consider what happen for all her possible actions.

To understand it, we see an example. please see the matrix at the bottom of page 2. That is the simplest game because it is a two-player and two-action game. The players are a row player and a column one. The row player chooses either U or D, and the column player chooses either L or R. In each cell, the left number represents the row player's payoff and the right one is the column player' one.

To find Nash equilibrium, each player needs to consider all the possible actions as I said before. So, first we consider what the row player should choose. To decide that, the row player considers when the column player chooses L. Then, the player compares between U and D, which means 3 and 2. Naturally 3 is greater than 2, and she chooses U under the assumption that the column player chooses L.

But, of course, The column player may choose R. So, the row player also consider which action is better when the column player chooses R, which means that the player comperes between 0 and -1. Then, naturally 0 is better than -1, and the row player chooses U when the column player chooses R.

In the same way, the column player also considers which action is better when the row player chooses U and D, respectively. The column player chooses L for U, and R for D.

As seen in the handout, (U, D) is the combination of the best actions for both the players. That is considered as Nash equilibrium in this game.

When each player has more possible actions, we need to consider in the same way to find the Nash equilibrium.

We move to page 3. The pure strategy can be extended into probability distributions over possible actions. That is called mixed actions or mixed strategies. The idea is very simple. You imagine the paper-rock-scissors game. When you play the game, the best action can be a randomized action over paper, rock, and scissors. The mixed action is just a representation of such behavior.

So the definition of mixed strategies is given as a probability distribution over pure strategies.

Move to page 4. Then, the payoff of mixed strategies is given as the top of page 4. Here s is a profile of players' actions. That is $s = (s_1, s_2, \ldots s_N,)$. Then, $m_i(s_i)$ is a probability that player i takes a pure action s_i in s. Note that s is considered over all small s in capital S. See the subscript of the summation. Once you understand the definition of this payoff, which is called expected utility, the idea of mixed Nash equilibrium is basically same as the pure strategy Nash equilibrium.

Look at page 5. Now let's consider how to find mixed strategies using an example. There are also two players in this game. It's a game that I (Ryu) and my wife play, and we're trying to figure out whether to go to the opera or go to a public lecture of the Dalai Lama. This is often referred as the battle of sexes.

From this payoff table, you can quickly see that the pure strategy Nash equilibrium of this game is that both the players go to the opera, and go to the Dalai Lama's lecture. Now, how can we find a mixed strategy Nash equilibrium? We see the movie file.

Look at page 6.
Ok. Let's try to get the Nash equilibria in this game.
To solve all the Nash equilibria, let p and 1-p be probabilities that the wife goes to Opera and Dalai Lama's lecture, respectively. Also let q and 1-q be probabilities that Ryu goes to Opera and Dalai Lama's lecture, respectively.

When Ryu goes to Opera, then his expected payoff is: $2\times p+0\times(1-p)=2p$ because he gets 2 with probability p and 0 with probability 1-p. For the same reason, when Ryu goes to the lecture, his expected payoff is: $0\times p+1\times(1-p)=1-p$.

Ryu compares between 2p and 1-p. If 2p > 1-p (which is equivalent to p>1/3), Ryu gets the higher payoff by going to the Opera. Then he should go to Opera. That means his probability to go to Opera, q, is equal to 1 (recall q is the probability he goes to Opera).
Otherwise but except p=1/3, he should go to the lecture. That is q = 0.
When p =1/3, he has no preference of the choice.

In the same way, her wife's best choice is also calculated. The result is she goes to the Opera when q > 1/2, otherwise she goes to the lecture except q=1/2. When q=1/2, she has no preference between them.

Now we visualize this result by making a plane with p and q.

Sorry for using a movie file. But the movie is helpful to understand how to solve mixed strategy Nash equilibrium. Since I prepare different qualities for your various internet environment, please choose either one.

Let's draw the graphs.
Since both the player can choose a probability between 0 and 1, we focus on the range on the plane. We take p = 1/3 on it. Then, this range is over 1/3, and the lower part is lower than 1/3. When p>1/3, q=1. Then the graph of Ryu's best response is drawn like this. In the same way, we can draw his best response graph when p < 1/3. Since he has no preference when p=1/3, he can choose any p between 0 and 1. So, graph is like this.

His wife's best response graph is also drawn like that. The intersections of the two graphs represent Nash equilibria. So there are three Nash equilibria. As you know, 1 and 2 are pure strategy Nash equilibrium, and the 3 represents mixed strategy Nash equilibrium. That is, Ryu goes to Opera with ½ & the lecture with ½, and the wife goes to Opera with 1/3 & the lecture with 2/3.

We summarize three Nash equilibria like this.

Please move to Page 7. This is a summary of the basic properties of Nash equilibrium. You try to make an example to describe each property.